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STATISTICAL MODEL OF NATURAL IMAGES

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ABSTRACT

In this article, we propose a statistical model of natural images in JPEG format. The image acquisition is composed of three principal stages. First, a RAW image is obtained from sensor of Digital Still Cameras (DSC). Then, the RAW image is subject to some post-acquisition processes such as demosaicking, white-balancing and γ-correction to improve its visual quality. Finally, the processed images goes through the JPEG compression process. For each step of image processing chain, a statistical study of pixels' properties is performed to finally obtain a model of Discrete Cosine Transform (DCT) coefficients distribution.

Index Terms— Statistical model, Natural image, JPEG format, DCT Coefficients.

1. INTRODUCTION

In the past decade, applications of computer vision for surveillance, security and safety people and systems have rapidly increased. The used images for these applications are usually compressed in JPEG format. The compression technology is a lossy process which help to reduce image size and can also strongly affect on the decision-making process. This explains why the study of image acquisition and processing chain from RAW format to JPEG format is of crucial importance.

Before display, a natural image acquired by a DSC undergoes some post-acquisition processes [1]. At this step, the initial RAW image which is obtained from Charge-Coupled Device sensor is not a full color image yet. The RAW image then goes through post-acquisition processes mainly composed of demosaicking, white balancing and γ-correction to obtain a TIFF image and improve its visual quality. Finally, the TIFF image goes through the JPEG compression process to reduce its size.

The organization of the paper is as follows. A state of the art is described in section 2. The statistical model of RAW image is explained in section 3. In section 4, we extend this model to a model of TIFF image. In section 5, we propose a model of DCT coefficients of JPEG image before the quantification. The section 6 presents some numerical results and the section 7 concludes the paper.

2. STATE OF THE ART

Over the past decades, there have been various studies on the distributions of the DCT coefficients for images. However, they have concentrated only on fitting the empirical data with a variety of well-known statistical distributions. Recently, Chang [3] has shown that the generalized Gamma model is more appropriate than the Laplacian model or the Generalized Gaussian model to characterize the DCT coefficients. However due to the lack of mathematical analysis, a theoretical explanation of these various models of the DCT coefficients distribution is still missing. Lam and Goodman [4] showed that a doubly stochastic model of the images provides us an insight into the distribution. This model combines a Gaussian model for the DCT coefficients when the block variance is ?xed and a model for block variance. However, it remains necessary to extend their work by studying the block variance to provide a universal model of DCT coef?cients. This model can be obtained from the study of the entire chain of image formation.

3. MODEL OF RAW IMAGE

A. Foi et al have studied and proposed an accurate model of a single RAW image. This RAW model is based on a Poissonian distribution modeling the photons sensing and a Gaussian distribution for the stationary disturbances in the output data. The normal approximation to the Poisson distribution gives a simplified model of the pixels RAW:

$$Z^c(m,n) \sim \mathcal{N}(\theta^c(m,n), S^c(m,n)) \tag{1}$$

where $Z^c(m,n)$ denotes the value of pixel at the location (m,n) for the color channel c (R, G, and B), θ^c and S^c are respectively its mean and variance. It can be noted that the mean and variance of the RAW pixel admit the following heteroscedastic relation:

$$S^c(m,n) = a \cdot \theta^c(m,n) + b \tag{2}$$

4. MODEL OF TIFF IMAGE

4.1. Normality and correlation in demosaicking and white balancing

In the scope of this paper, we study only the bilinear interpolation for the demosaicking algorithm and the Gray World for the white balancing algorithm.

For each color channel, the bilinear interpolation [1] calculates the missing value as a weighted mean of its neighbors which can be written as a linear filtering $Z^c_{DM} = h^c \circledast Z^c$ where h^c is the linear filter for color channel c and $\circledast$ denotes the 2-D convolution. Due to the linear property of the Gaussian distribution, the distribution of the pixel after the bilinear interpolation can be expressed as:

$$Z^c_{DM}(m,n) \sim \mathcal{N}(\theta^c_{DM}(m,n), S^c_{DM}(m,n)) \tag{3}$$

where $\theta^c_{DM} = h^c \circledast \theta^c$ and $S^c_{DM} = (h^c)^2 \circledast S^c$. As the missing values are interpolated from these neighbors, a pixel within a channel is correlated to their neighbors. This correlation is periodic due to the periodic pattern of the Bayer CFA.

The white balancing process is implemented after the demosaicking process to correct illuminant light color modification. The Gray World algorithm [1] assumes that the average intensities of the red, green and blue channels should be equal. Therefore, the red and blue channel gains (the green channel is usually left unchanged) are calculated as:

$$C_{gain} = \mu^g_{DM} / \mu^c_{DM} \tag{4}$$

where μ^c_{DM} is the average intensity value for the color channel c. So the value of the red and the blue channel are modified by:

$$Z^c_{WB}(m,n) = C_{gain} \cdot Z^c_{DM}(m,n) \tag{5}$$

For most of digital images, the number of pixels is sufficiently large to approximate $\mu^c_{DM} = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} Z^c_{DM}(m,n)$ by $\overline{\theta}^c_{DM} = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} \theta^c_{DM}(m,n)$. Using this approximation, the distribution of the pixel after the white balancing process can be expressed as:

$$Z^c_{WB}(m,n) \sim \mathcal{N}(\theta^c_{WB}(m,n), S^c_{WB}(m,n)) \tag{6}$$

where $\theta^c_{WB} = C_{gain} \cdot \theta^c_{DM}$ and $S^c_{WB} = C^2_{gain} \cdot S^c_{DM}$. Let notice that after the white balancing process, pixels distribution remains Gaussian with a correlation between neighbors due to demosaicking.

4.2. Non-normality in γ-correction

Finally, the γ-correction process necessary for contrast display purpose, is given as:

$$Z^c_{GM}(m,n) = |Z^c_{WB}(m,n)|^{\frac{1}{\gamma}} \tag{7}$$

This process involve a non-linear input-output mapping after which pixels are not Gaussian anymore. One can note that the pixel Z^c_{WB} is distributed as a Gaussian random variable, hence, $|Z^c_{WB}|$ has a folded normal distribution. By applying the change of variables theorem, the distribution of the pixel Z^c_{GM} after γ-correction can finally be written:

$$f(Z^c_{GM}) = \frac{\gamma (Z^c_{GM})^{\gamma-1}}{\sqrt{2\pi S^c_{WB}}}\Bigg[\exp\Big(-\frac{((Z^c_{GM})^\gamma - \theta^c_{WB})^2}{2S^c_{WB}}\Big) + \exp\Big(-\frac{((Z^c_{GM})^\gamma + \theta^c_{WB})^2}{2S^c_{WB}}\Big)\Bigg] \tag{8}$$

$f(X)$ denotes a shortcut of the probability density function (pdf) of a random variable X.

5. DOUBLY STOCHASTIC MODEL OF DCT COEFFICIENTS

In the JPEG compression, the image should be converted from RGB into a different color space called YCbCr. This transformation doesn't change a lot the distribution of TIFF pixels due to the ponderation of three color channels RGB but it's no use characterizing it here. Then, each channel (Y, Cb, Cr) must be divided into 8×8 blocks and each block is converted to a frequency domain representation using the DCT operation :

$$I_{ij} = \frac{1}{4} T_i T_j \sum_{m=0}^{7}\sum_{n=0}^{7} Z^t(m,n) \cdot \cos\Big(\frac{(2m+1)i\pi}{16}\Big)\cos\Big(\frac{(2n+1)j\pi}{16}\Big) \tag{9}$$

where Z^t denotes the pixel for the color channel t (Y, Cb, Cr) and

$$T_k = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } k = 0 \\ 1 & \text{for } k > 0 \end{cases}$$

Assuming Z^t pixels are identically distributed within a block, Lam and Goodman [4] confirmed that the model of DCT coefficients should be performed as a doubly stochastic model due to the variability of pixel value within a block. Using conditional probability, the global model of DCT coefficients can be expressed as:

$$f(I_{ij}) = \int_0^\infty f(I_{ij}|\sigma^2) f(\sigma^2) d(\sigma^2) \tag{10}$$

where σ^2 denotes the block variance.

Given a constant block variance, the DCT coefficients can be approximatively normally distributed by using the central limit theorem version for correlated random variables detailed in [5], which provides a result on asymptotic distribution of a sum of correlated variables. In our case, the variables are spatially correlated, which is a special case of the correlation studied in [5]. As the three first moments of each element are obviously finite, the distribution can be expressed as :

$$f(I_{ij}|\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{I_{ij}^2}{2\sigma^2}} \tag{11}$$

Equations (11) and (12) highlight the crucial importance to analyze the distribution of the block variance $f(\sigma^2)$ in order to calculate the global distribution of DCT coefficients.

5.1. Block variance : Gamma model

The variance of block k can be expressed as :

$$\sigma_k^2 = \frac{1}{63}\sum_{m=0}^{7}\sum_{n=0}^{7}\left(Z_k^t(m,n) - \overline{Z_k^t}\right)^2 \quad (12)$$

where $\overline{Z_k^t}$ is the mean of pixels within block k, which is also a random variable. We have :

$$Z_k^t(m,n) - \overline{Z_k^t} = \frac{1}{64}\sum_{u=0}^{7}\sum_{v=0}^{7}\left(Z_k^t(m,n) - Z_k^t(u,v)\right) \quad (13)$$

By using the version of the central limit theorem from [5], $Z_k^t(m,n) - \overline{Z_k^t}$ can be approximated as a zero-mean Gaussian random variable. The square of a Gaussian variable is distributed as Chi-squared variable with one degree of freedom. Furthermore, a Chi-squared variable weighted by a positive constant is distributed as Gamma random variable $\mathcal{G}(\alpha, \beta_i)$ with shape parameter $\alpha = 1/2$. As a result, the block variance σ_k^2 can be considered as a sum of correlated Gamma variables.

The exact distribution of the sum of arbitrarily correlated Gamma random variables is explicitly given in [6]. Let definite $X = \sum_{i=1}^{n} X_i$ where $X_i \sim \mathcal{G}(\alpha, \beta_i))$ and let ρ_{ij} be the correlation coefficient between X_i and X_j. It is shown that the moment-generating function (MGF) of X is given by:

$$\mathcal{M}_X(s) = \prod_{i=1}^{n}(1 - s\lambda_i)^{-\alpha} \quad (14)$$

where λ_i, $i = 1, \ldots, n$, are the eigenvalues of the matrix $A = DM$, D is the $n \times n$ diagonal matrix with the entries β_i and M is the $n \times n$ positive definite covariance matrix.

The MGF of X is in a similar form as the MGF of the sum of independent Gamma random variables $Y = \sum_{i=1}^{n} Y_i$ where $Y_i \sim \mathcal{G}(\alpha, \lambda_i)$. We use the moment matching method to approximate the complicated distribution of Y by a Gamma distribution. The two first positive moments of Y are:

$$\mathbb{E}(Y) = \alpha\sum_{i=1}^{n}\lambda_i \qquad Var(Y) = \alpha\sum_{i=1}^{n}\lambda_i^2 \quad (15)$$

On the other hand, the moments of the Gamma distribution with parameters α^* and β^* are defined by :

$$\mathbb{E}(Y) = \alpha^*\beta^* \qquad Var(Y) = \alpha^*(\beta^*)^2 \quad (16)$$

The approximating Gamma distribution can be matched as

$$\alpha^* = \alpha\frac{(\sum_{i=1}^{n}\lambda_i)^2}{\sum_{i=1}^{n}\lambda_i^2} = \frac{(\sum_{i=1}^{n}\lambda_i)^2}{2\sum_{i=1}^{n}\lambda_i^2} \qquad \beta^* = \frac{\sum_{i=1}^{n}\lambda_i^2}{\sum_{i=1}^{n}\lambda_i} \quad (17)$$

As a result, we can model the block variance with the Gamma distribution $\mathcal{G}(\alpha^*, \beta^*)$.

5.2. Global model of DCT coefficients

As a result, the global distribution of DCT coefficients can be expressed as :

$$\begin{aligned} f(I_{ij}) &= \int_0^\infty \left(\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{I_{ij}^2}{2\sigma^2}}\right)\left(\frac{(\sigma^2)^{\alpha^*-1}}{(\beta^*)^{\alpha^*}\Gamma(\alpha^*)}e^{-\frac{\sigma^2}{\beta^*}}\right)d(\sigma^2) \\ &= \frac{1}{\sqrt{2\pi}(\beta^*)^{\alpha^*}\Gamma(\alpha^*)}\int_0^\infty e^{-\left(\frac{\sigma^2}{\beta^*}+\frac{I_{ij}^2}{2\sigma^2}\right)}(\sigma^2)^{\alpha^*-\frac{3}{2}}d(\sigma^2) \end{aligned} \quad (18)$$

From [7], the modified Bessel function is given by

$$K_\nu(xz) = \frac{z^\nu}{2}\int_0^\infty e^{-\frac{x}{2}\left(t+\frac{z^2}{t}\right)}t^{-\nu-1}dt \quad (19)$$

By replacing $x = \frac{2}{\beta^*}$, $z = |I_{ij}|\sqrt{\frac{\beta^*}{2}}$, $\nu = \frac{1}{2} - \alpha^*$, we obtain

$$\begin{aligned} K_{\frac{1}{2}-\alpha^*}\left(|I_{ij}|\sqrt{\frac{2}{\beta^*}}\right) &= \frac{1}{2}\left(|I_{ij}|\sqrt{\frac{\beta^*}{2}}\right)^{\frac{1}{2}-\alpha^*} \\ &\cdot\int_0^\infty e^{-\left(\frac{\sigma^2}{\beta^*}+\frac{I_{ij}^2}{2\sigma^2}\right)}(\sigma^2)^{\alpha^*-\frac{3}{2}}d(\sigma^2) \end{aligned} \quad (20)$$

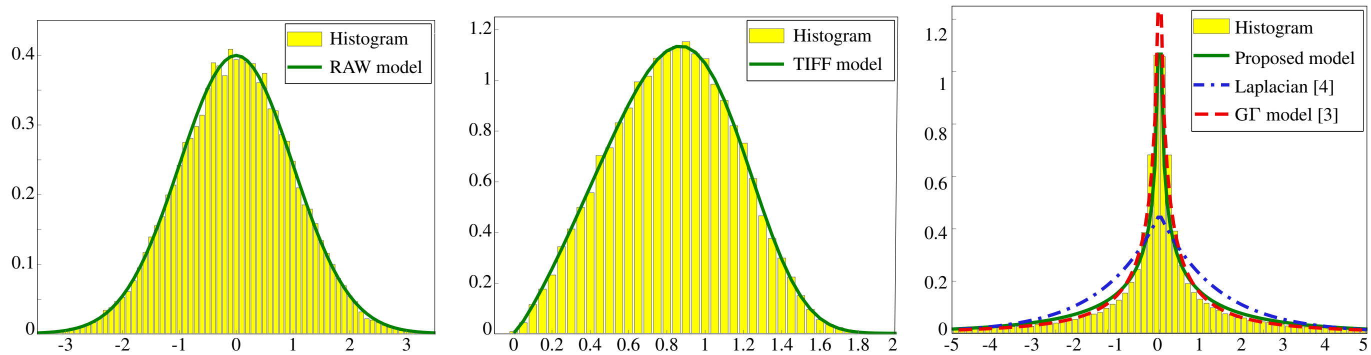
From (19) and (21) and remarking that $K_\nu(x) = K_{-\nu}(x)$, we derive

$$f(I_{ij}) = \frac{\sqrt{2}\left(|I_{ij}|\sqrt{\frac{\beta^*}{2}}\right)^{\alpha^*-\frac{1}{2}}}{\sqrt{\pi}(\beta^*)^{\alpha^*}\Gamma(\alpha^*)}K_{\alpha^*-\frac{1}{2}}\left(|I_{ij}|\sqrt{\frac{2}{\beta^*}}\right) \quad (21)$$

It is noted that the Gaussian model and the Laplacian model are two special cases of this one. Indeed, when $\alpha^* = 1$, the Gamma distribution becomes the exponential distribution. Lam and Goodman [4] demonstrated how to obtain the Laplacian model from the exponential distribution of the block variance. When $\alpha^* \to \infty$, the block variance tends to zero, consequently the DCT coefficients tend to be distributed as Gaussian.

6. NUMERICAL RESULTS

We performed experiments with a subset of the Dresden Image Database [8] to verify the models presented above. We chose original RAW images and converted them into uncompressed TIFF image using Dcraw (with parameters -D -4 -T -j -v). This experiment was conducted along the image processing chain. We showed that the fit of the models to the data is nearly perfect in Figure 1. In addition, we showed that the Laplacian model, which remains a popular choice in the literature, is not adequate anymore to model the DCT coefficients distribution. However, the Figure 1c doesn't show the difference between the proposed model and the Generalized Gamma model. Consequently, we conducted the χ^2



(a) Distribution of RAW pixels. (b) Distribution of TIFF pixels. (c) Distribution of DCT coefficients.

Fig. 1: Example of proposed model applied for natural image content estimation and noise whitening.

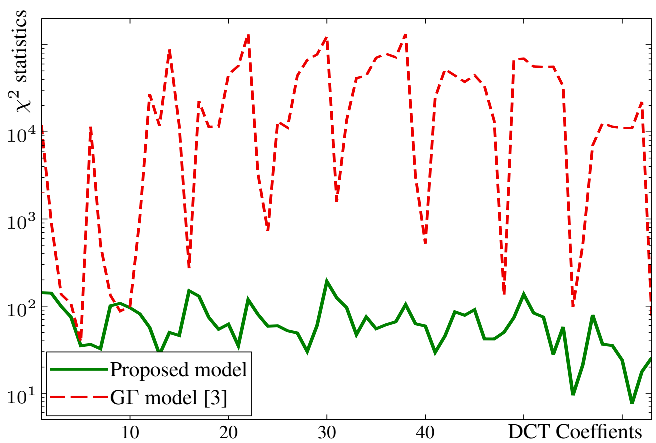


Fig. 2: χ^2 GOF test results of GΓ and proposed model.

goodness-of-fit (GOF) test to bring them into comparison, where the parameters of the proposed model are estimated using the Maximum Likelihood Estimation approach. The Figure 2 shows the mean result of each DCT coefficient over 100 images and that the proposed model is more desirable than the Generalized Gamma model.

7. CONCLUSION

In this paper, we have given a comprehensive insight about the image processing chain, from RAW format to JPEG format. We have proposed a global model of DCT coefficients and given an analytic justification by using a doubly stochastic model. We have shown that the model proposed for DCT coefficients is more appropriate than the Generalized Gamma model. It is noted that a Laplacian distribution, which remains a popular choice in the literature, is a special case of the model proposed.

Further research could look into the distribution of the quantized DCT coefficients so that we can build a global model of a JPEG image, which could give us a better insight into JPEG compression algorithm. This global model could be useful and applied in various domains in the future.

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